Spiral cylindrique sans courbes terminales

Développement excentrique et anisochronisme en position horizontale

Déformations planes

Caractéristiques du spiral

Référence :E:\Résonateur (TA)\Data\Bal_spiral cylindrique (ex num).mcd(R)

Référence :E:\Résonateur (TA)\Data\Définition Atan.mcd(R)

Dimensions $\quad ép = 0.09\,mm \quad ha = 0.334\,mm \quad S = 0.03\,mm^2 \quad R_0 = 5\,mm$

Elinvar $\quad \rho_s = 8\times10^3\,kg\cdot m^{-3} \quad E = 1.7\times10^{11}\,Pa \quad G = 6.538\times10^{10}\,Pa$

Partie cylindrique $\quad n_s := 10.15 \quad \psi_0 := n_s\cdot 360\cdot deg \quad \psi_0 = 3.654\times10^3\,deg \quad L := R_0\cdot\psi_0 \quad L = 318.872\,mm$

$r_s(\alpha) := R_0 \quad s(\alpha) := R_0\cdot\alpha \quad x_{0s}(\alpha) := R_0\cdot cos(\alpha) \quad y_{0s}(\alpha) := R_0\cdot sin(\alpha)$

Positions du piton $\quad r_P := R_0 \quad \alpha_P := 0 \quad x_P = 5\,mm \quad y_P = 0\,mm$

Position du point d'attache à la virole $\quad r_V := R_0 \quad \alpha_V(\theta) := \psi_0 + \theta \quad x_V(\theta) := r_V\cdot cos(\alpha_V(\theta)) \quad y_V(\theta) := r_V\cdot sin(\alpha_V(\theta))$

Amplitude stationnaire du balancier $\quad \theta_0 := 270\cdot deg$

Contrainte maximum

Référence :E:\Résonateur (TA)\Tables\Modules J, I et W des barres élastiques.mcd(R)

$$I_{33} := I_{f\_rect}(ép, ha) \quad W_{f3} := W_{f\_rect}(ép, ha) \quad \sigma_{max} := \frac{E\cdot I_{33}}{L\cdot W_{f3}}\cdot\theta_0 \quad \sigma_{max} = 113.054\,\frac{N}{mm^2}$$

Première approximation de la déformée du spiral

$$\varphi_0(\alpha) := \alpha + \frac{\pi}{2} \quad z_P := x_P + i\cdot y_P \quad z_1(\theta,\alpha) := z_P + R_0\cdot\int_0^{\alpha} i\cdot exp(i\cdot\alpha')\cdot exp\left(i\cdot\theta\cdot\frac{\alpha'}{\psi_0}\right)d\alpha'$$

$$z_0(\alpha) := R_0\cdot exp(i\cdot\alpha) \quad z_1(\theta,\alpha) := z_P + R_0\cdot\frac{\psi_0}{\psi_0+\theta}\cdot\left(exp\left(i\cdot\alpha\cdot\frac{\psi_0+\theta}{\psi_0}\right) - 1\right)$$

Graphe de la déformation

Forme naturelle

$$n := 20\cdot partentière(n_s) + 1 \quad i := 0..n-1 \quad \Delta\alpha := \frac{\psi_0}{n-1} \quad \alpha_i := i\cdot\Delta\alpha$$

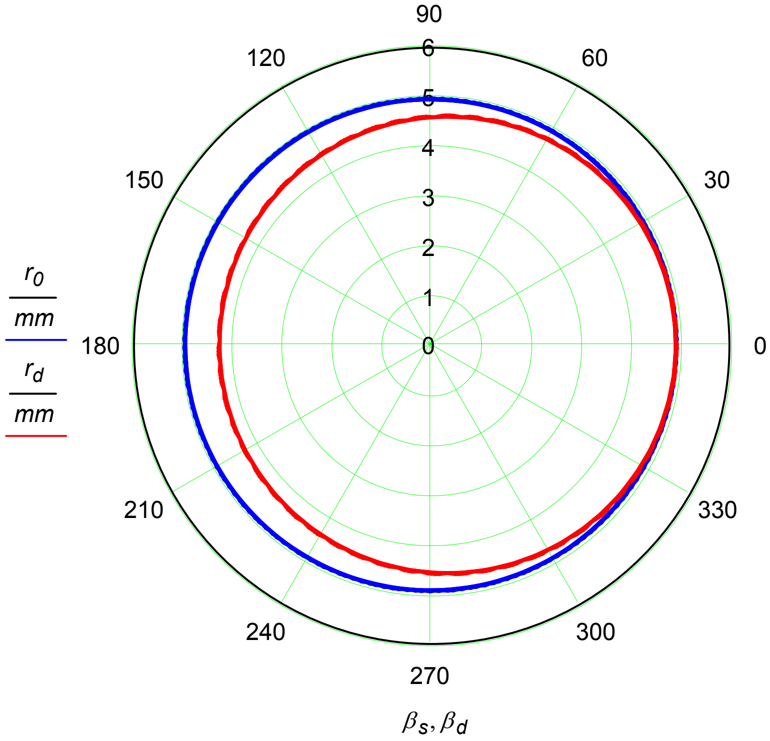
$$x_{0_i} := x_{0s}(\alpha_i) \quad y_{0_i} := y_{0s}(\alpha_i) \quad r_0 := \overrightarrow{\sqrt{x_0^2 + y_0^2}} \quad \beta_s := \overrightarrow{Atan(x_0, y_0)}$$

Déformée

$$z_{d_i} := z_1(\theta_0, \alpha_i) \quad n_{pt} := dernier(z_d) \quad x_d := Re(z_d) \quad y_d := Im(z_d) \quad r_d := \overrightarrow{|z_d|} \quad r_{d_{npt}} = 4.938\,mm$$

$$\beta_d := \overrightarrow{Atan(x_d, y_d)} \quad \beta_{d_0} = 0\,deg \quad \beta_{d_{npt}} = 326.347\,deg$$

$$mod(\alpha_V(\theta_0), 2\cdot\pi) = 324\,deg$$



$mod(\psi_0, 2\cdot\pi) = 54\,deg$

$r_P = 5\,mm$

$r_V = 5\,mm$

$\alpha_V(0) = 3.654\times 10^3\,deg$

$x_V(\theta_0) = 4.045\,mm$

$y_V(\theta_0) = -2.939\,mm$

$\Delta x_V := x_{d_{npt}} - x_V(\theta_0)$

$\Delta x_V = 0.066\,mm$

$\Delta y_V := y_{d_{npt}} - y_V(\theta_0)$

$\Delta y_V = 0.202\,mm$

Déplacement de la virole libre

$$\mathbf{\Delta_1}(\theta) := \frac{i\cdot\theta}{\psi_0}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot exp\left(i\cdot\theta\cdot\frac{\alpha}{\psi_0}\right)d\alpha \qquad \mathbf{\Delta_1}(\theta) := \frac{\theta}{\psi_0+\theta}\cdot R_0\cdot\left[exp\left[i\cdot(\psi_0+\theta)\right]-1\right]$$

$$u_1(\theta) := Re(\mathbf{\Delta_1}(\theta)) \qquad v_1(\theta) := Im(\mathbf{\Delta_1}(\theta)) \qquad u_1(\theta_0) = -0.066\,mm \qquad v_1(\theta_0) = -0.202\,mm$$

Calcul des réactions

$$\xi_{0s} := \frac{R_0}{\psi_0}\cdot sin(\psi_0) \qquad \eta_{0s} := \frac{R_0}{\psi_0}\cdot(1-cos(\psi_0))$$

$$q2_{0s} := \frac{R_0^2}{2\cdot\psi_0}\cdot(\psi_0 - cos(\psi_0)\cdot sin(\psi_0)) \qquad p2_{0s} := \frac{R_0^2}{2\cdot\psi_0}\cdot(\psi_0 + cos(\psi_0)\cdot sin(\psi_0)) \qquad k_{0s} := \frac{R_0^2}{2\cdot\psi_0}\cdot sin(\psi_0)^2$$

$$\xi_{0s} = 0.063\,mm \qquad \eta_{0s} = 0.032\,mm \qquad q2_{0s} = 12.407\,mm^2 \qquad p2_{0s} = 12.593\,mm^2 \qquad k_{0s} = 0.128\,mm^2$$

$$\mathbf{S_0} := \frac{L}{E\cdot I_{33}}\cdot\begin{pmatrix} q2_{0s}-\eta_{0s}^2 & \eta_{0s}\cdot\xi_{0s}-k_{0s} \\ \eta_{0s}\cdot\xi_{0s}-k_{0s} & p2_{0s}-\xi_{0s}^2 \end{pmatrix} \qquad \mathbf{R'}(\theta) := \mathbf{S_0}^{-1}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta)\end{pmatrix} \qquad \mathbf{R'}(\theta_0) = \begin{pmatrix} -5.914\times10^{-5} \\ -1.746\times10^{-4}\end{pmatrix}N$$

$$\left|\mathbf{R'}(\theta_0)\right| = 1.843\times10^{-4}\,N$$

Approximations

$$\sigma2 := \frac{1}{\psi_0}\cdot\int_0^{\psi_0} z_0(\alpha)\cdot\overline{z_0(\alpha)}\,d\alpha \qquad \sigma2 := R_0^2 \qquad \frac{\sigma2}{2} = 12.5\,mm^2$$

$$R'(\theta) := \frac{E\cdot I_{33}}{L}\cdot\frac{2}{\sigma2}\cdot\begin{pmatrix} u_1(\theta) \\ v_1(\theta)\end{pmatrix} \qquad R'(\theta_0) = \begin{pmatrix} -5.693\times10^{-5} \\ -1.752\times10^{-4}\end{pmatrix}N \qquad \left|R'(\theta_0)\right| = 1.842\times10^{-4}\,N$$

Deuxième approximation de la déformée du spiral

$x_1(\theta,\alpha) := Re(z_1(\theta,\alpha))$ $\quad y_1(\theta,\alpha) := Im(z_1(\theta,\alpha))$ $\quad R'_x(\theta) := \mathbf{R'}(\theta)_0$ $\quad R'_y(\theta) := \mathbf{R'}(\theta)_1$

$s\xi_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha')\cdot R_0\, d\alpha'$ $\quad \xi_{1s}(\theta) := \frac{1}{L}\cdot s\xi_1(\theta,\psi_0)$ $\quad \xi_{1s}(\theta_0) = 0.304\,mm$

$s\eta_1(\theta,\alpha) := \int_0^{\alpha} y_1(\theta,\alpha')\cdot R_0\, d\alpha'$ $\quad \eta_{1s}(\theta) := \frac{1}{L}\cdot s\eta_1(\theta,\psi_0)$ $\quad \eta_{1s}(\theta_0) = 0.013\,mm$

$sp2_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha')^2\cdot R_0\, d\alpha'$ $\quad p2_1(\theta) := \frac{1}{L}\cdot sp2_1(\theta,\psi_0)$ $\quad p2_1(\theta_0) = 10.775\,mm^2$

$sq2_1(\theta,\alpha) := \int_0^{\alpha} y_1(\theta,\alpha')^2\cdot R_0\, d\alpha'$ $\quad q2_1(\theta) := \frac{1}{L}\cdot sq2_1(\theta,\psi_0)$ $\quad q2_1(\theta_0) = 10.994\,mm^2$

$sk_1(\theta,\alpha) := \int_0^{\alpha} x_1(\theta,\alpha')\cdot y_1(\theta,\alpha')\cdot R_0\, d\alpha'$ $\quad k_1(\theta) := \frac{1}{L}\cdot sk_1(\theta,\psi_0)$ $\quad k_1(\theta_0) = 0.117\,mm^2$

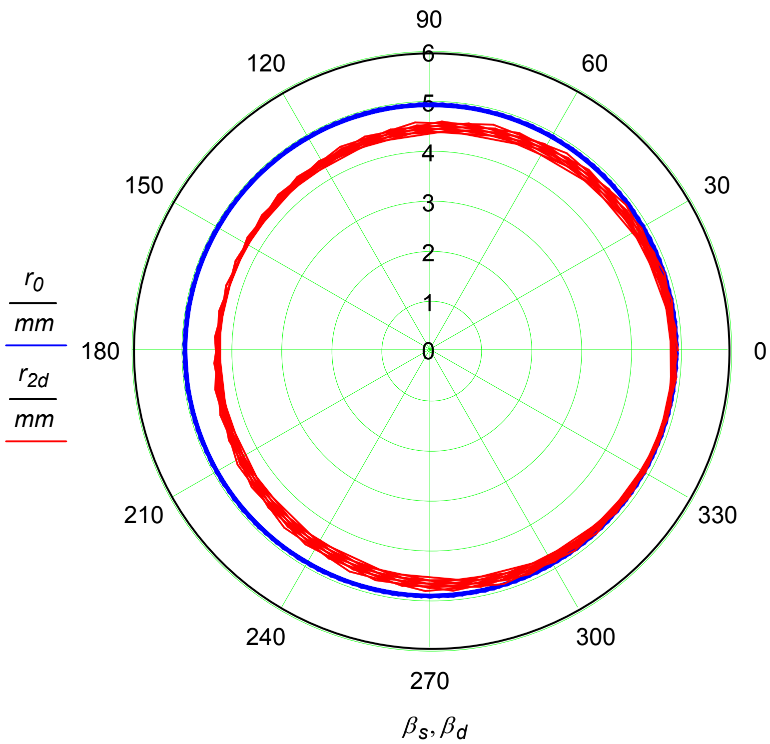
$$\mathbf{S_1}(\theta,\alpha) := \frac{1}{E\cdot I_{33}}\cdot\begin{pmatrix} -y_1(\theta,\alpha)\cdot s\eta_1(\theta,\alpha) + sq2_1(\theta,\alpha) & y_1(\theta,\alpha)\cdot s\xi_1(\theta,\alpha) - sk_1(\theta,\alpha) \\ x_1(\theta,\alpha)\cdot s\eta_1(\theta,\alpha) - sk_1(\theta,\alpha) & -x_1(\theta,\alpha)\cdot s\xi_1(\theta,\alpha) + sp2_1(\theta,\alpha) \end{pmatrix}$$

$\mathbf{R'}(\theta) := \mathbf{S_1}(\theta,\psi_0)^{-1}\cdot\begin{pmatrix} x_V(\theta) - x_1(\theta,\psi_0) \\ y_V(\theta) - y_1(\theta,\psi_0)\end{pmatrix}$ $\quad \mathbf{R'}(\theta_0) = \begin{pmatrix} -8.436\times 10^{-5} \\ -2.305\times 10^{-4}\end{pmatrix} N$ $\quad \mathbf{\Delta z_s}(\theta,\alpha) := \mathbf{S_1}(\theta,\alpha)\cdot\mathbf{R'}(\theta)$

$\Delta z_1(\theta,\alpha) := \mathbf{\Delta z_s}(\theta,\alpha)_0 + i\cdot\mathbf{\Delta z_s}(\theta,\alpha)_1$ $\quad z_2(\theta,\alpha) := z_1(\theta,\alpha) + \Delta z_1(\theta,\alpha)$

$z_{2d_i} := z_2(\theta_0,\alpha_i)$ $\quad n_{pt} := dernier(z_d)$ $\quad x_{2d} := Re(z_{2d})$ $\quad y_{2d} := Im(z_{2d})$ $\quad r_{2d} := \overrightarrow{|z_{2d}|}$ $\quad r_{2d_{npt}} = 5\,mm$

$\beta_d := \overrightarrow{Atan(x_{2d}, y_{2d})}$ $\quad \beta_{d_0} = 0\,deg$ $\quad \beta_{d_{npt}} = 324\,deg$ $\quad mod(\alpha_V(\theta_0), 2\cdot\pi) = 324\,deg$



$mod(\psi_0, 2\cdot\pi) = 54\,deg$

$r_P = 5\,mm$

$r_V = 5\,mm$

$\alpha_V(0) = 3.654\times 10^3\,deg$

$x_V(\theta_0) = 4.045\,mm$

$y_V(\theta_0) = -2.939\,mm$

$\Delta x_V := x_{2d_{npt}} - x_V(\theta_0)$

$\Delta x_V = 0\,mm$

$\Delta y_V := y_{2d_{npt}} - y_V(\theta_0)$

$\Delta y_V = 0\,mm$

Perturbation de période - spiral non déformé en position de repos

$$\Delta_1(\theta) := \frac{i\cdot\theta}{L}\cdot R_0^{\,2}\cdot\int_0^{\psi_0} exp\left[i\cdot\left(\theta\cdot\frac{R_0}{L}+1\right)\cdot\alpha\right]d\alpha \qquad \Delta_1(\theta) := \frac{\theta}{\psi_0+\theta}\cdot R_0\cdot\left[exp\left[i\cdot\left(\psi_0+\theta\right)\right]-1\right]$$

$$X(\theta) := \frac{\left(\left|\Delta_1(\theta)\right|\right)^2}{\sigma 2} \qquad X(\theta) := \frac{2}{\sigma 2}\cdot\left(\frac{\theta}{\psi_0+\theta}\cdot R_0\right)^2\cdot\left(1-cos\left(\psi_0+\theta\right)\right) \qquad \gamma(\theta) := \frac{d}{d\theta}X(\theta)$$

$$\theta(\varphi) := \theta_0\cdot cos(\varphi) \qquad Delta\left(\theta_0\right) := \frac{-1}{2\cdot\pi\cdot\theta_0}\cdot\int_0^{2\cdot\pi}\gamma\left(\theta_0\cdot cos(\varphi)\right)\cdot cos(\varphi)\,d\varphi$$

$$\mu\left(\theta_0\right) := -86400\cdot Delta\left(\theta_0\right) \qquad \boxed{\mu\left(\theta_0\right) = 24.581} \qquad \boxed{\mu(180\cdot deg) = 75.453}$$

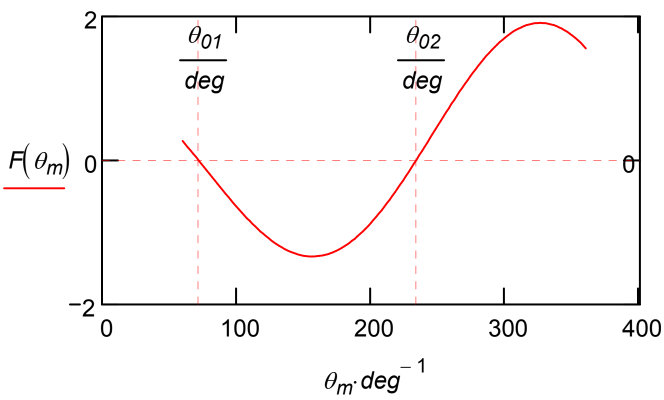
Approximation

$$F(x) := J0(x) - x\cdot J1(x) \qquad F\left(\theta_0\right) = 1.061 \qquad F'(x) := \frac{d}{dx}F(x)$$

$$x := 100\cdot deg \qquad \theta_{01} := racine(F(x),x) \qquad \theta_{01} = 72\,deg \qquad \theta_{m1} := racine(F'(x),x) \qquad \theta_{m1} = 156.7\,deg$$

$$x := 300\cdot deg \qquad \theta_{02} := racine(F(x),x) \qquad \theta_{02} = 233.7\,deg \qquad \theta_{m2} := racine(F'(x),x) \qquad \theta_{m2} = 326.1\,deg$$

$$\theta_m := 60\cdot deg, 62\cdot deg .. 360\cdot deg$$

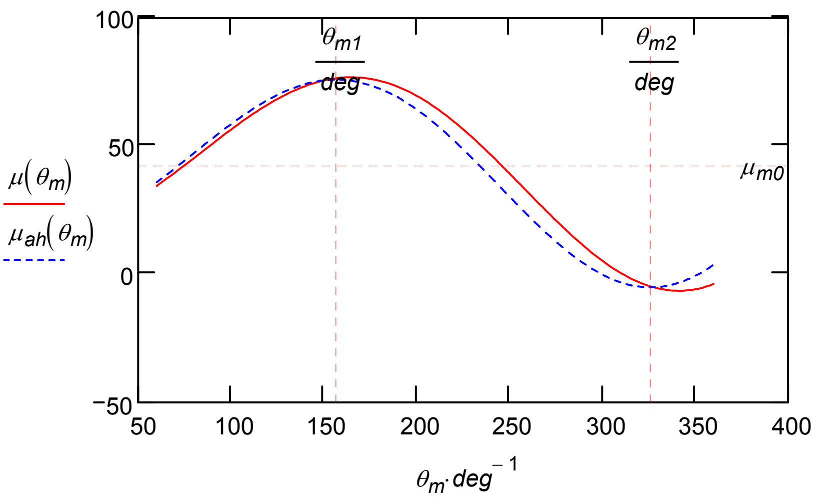


$$\delta_{ah}\left(\theta_0\right) := \frac{2}{\psi_0^{\,2}}\cdot\left(-1+F\left(\theta_0\right)\cdot cos\left(\psi_0\right)\right)$$

$$\mu_{ah}\left(\theta_0\right) := -86400\cdot\delta_{ah}\left(\theta_0\right)$$

$$\boxed{\mu_{ah}\left(\theta_0\right) = 15.98}$$

$$\boxed{\mu_{ah}(220\cdot deg) = 52.235}$$



$$\mu_{m0} := \frac{86400\cdot 2}{\psi_0^{\,2}}$$

$$\mu_{m1} := -86400\cdot Delta\left(\theta_{m1}\right)$$

$$\mu_{m1} = 76.423$$

$$\mu_{m2} := -86400\cdot Delta\left(\theta_{m2}\right)$$

$$\mu_{m2} = -4.799$$